

Radiative Seesaw Model with Degenerate Majorana Dark Matter

Takaaki Nomura,^{1,*} Hiroshi Okada,^{2,†} and Yuta Orikasa^{1,3,‡}

¹*School of Physics, KIAS, Seoul 130-722, Korea*

²*Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan 300*

³*Department of Physics and Astronomy,
Seoul National University, Seoul 151-742, Korea*

(Dated: March 16, 2016)

We study a three loop induced neutrino mass model with exotic vector-like isospin doublet leptons which contain a dark matter candidate. Then we explore lepton flavor violations, and dark matter physics in co-annihilation system. In this paper the nearly degenerate Majorana fermion dark matter can naturally be achieved at the two loop level, while the mass splitting can be larger than $\mathcal{O}(200)$ keV which is required from the constraint of the direct detection search with spin independent inelastic scattering through Z boson portal. As a result a monochromatic photon excess, its threshold energy is greater than $\mathcal{O}(200)$ keV, is predicted in our model that could be measured through indirect detection experiments such as INTEGRAL.

PACS numbers:

*Electronic address: nomura@kias.re.kr

†Electronic address: macokada3hiroshi@gmail.com

‡Electronic address: orikasa@kias.re.kr

I. INTRODUCTION

If the active neutrino could be related to new physics beyond the standard model (SM) such as dark matter (DM) candidate within TeV scale, it could be verifiable models through various experiments in the near future. In this sense, radiative seesaw models could be one of the natural realization of the tiny neutrino mass at TeV energy scale involving DM. Along this line of ideas, a vast paper has been arisen in Refs. [1–105].

In this paper, a three loop induced neutrino mass model is studied where we analyze lepton flavor violations, and dark matter physics in co-annihilation system. To generate active neutrino mass, we introduce extra $SU(2)_L$ doublet leptons whose neutral component can be a DM candidate. For each doublet, we have two neutral Majorana fermions with degenerate mass at the tree level since the extra leptons are introduced as vector-like, which is highly constrained by direct detection search of DM due to Z boson exchanging process. The splitting of neutral fermion mass degeneracy can naturally be achieved at the two loop level by introducing extra isospin singlet charged scalar fields. As a result, a monochromatic photon excess could be measured through indirect detection experiments such as INTEGRAL [106] at 511 keV photon emission energy where its threshold energy is greater than $\mathcal{O}(200)$ keV [107]. Here the lower limit of the energy comes from the constraint of the direct detection search with spin independent inelastic scattering through Z boson portal. Also we discuss the way to increase the mass difference between these degenerated masses at the two loop level through Zee-Babu type diagram. Since this mass is proportional to the charged lepton masses, the natural scale is the active neutrino masses $\mathcal{O}(0.1 - 1)$ eV, which is ruled out by the direct detection searches. To evade the problem, we introduce additional isospin singlet charged bosons with the same quantum number as the original field contents. Then the degenerated mass scale can achieve the order $\mathcal{O}(200)$ keV. However, the muon anomalous magnetic moment is also negatively enhanced by this effect. Here we also discuss the way to evade this constraint.

This paper is organized as follows. In Sec. II, we show our model building including Yukawa and Higgs sector. In Sec. III we discuss phenomenology of the model such that mass formulations, and analysis of the DM physics. We conclude and discuss in Sec. IV.

Fermion	L_L	e_R	L'
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, -1/2)$	$(\mathbf{1}, -1)$	$(\mathbf{2}, -1/2)$
Z_2	+	+	-

TABLE I: Lepton sector, where the three flavor index for each field L_L , e_R and L' is abbreviated.

Boson	Φ	h_i^+	k_i^{++}	S
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, 1/2)$	$(\mathbf{1}, 1)$	$(\mathbf{1}, 2)$	$(\mathbf{1}, 0)$
Z_2	+	-	+	-

TABLE II: Boson sector: i runs 1 to N.

II. THE MODEL

In this section, we explain our model. For the fermion sector, we introduce $SU(2)_L$ doublet exotic leptons (L') with three flavors to the SM leptons (L_L, e_R), as we summarize in Tab. I. For the boson sector, we introduce a $SU(2)_L$ singlet inert scalar (S), $SU(2)_L$ isospin singlet singly charged scalars (h_i^\pm) and doubly-charged scalars ($k_i^{\pm\pm}$) to the SM Higgs boson (Φ), as we summarize in Tab. II. Z_2 symmetry plays an role in our radiative neutrino model at the three loop, and assures the stability of the dark matter candidates L' and/or S . Φ develops a VEV (denoted by $v/\sqrt{2}$) after breaking the $SU(2)_L$ symmetry.

The renormalizable Lagrangian for Yukawa sector and scalar potential under these assignments are relevantly given by

$$-\mathcal{L}_Y = (y_\ell)_{ij} \bar{L}_{L_i} \Phi_L e_{R_j} + (f_R)_{ij} \bar{L}_{L_i} L'_{R_j} S + \sum_{a=1}^N \left((f_L)_{ij}^a \bar{L}'_{L_i} i\tau_2 L_{L_j}^C h_a^- + g_{ij}^a \bar{e}_i^C e_j k_a^{++} \right) + (M_E)_i \bar{L}'_{L_i} L'_{L_i} + \text{c.c.}, \quad (\text{II.1})$$

$$\begin{aligned} \mathcal{V} = & -m_\Phi^2 |\Phi|^2 - \sum_{a=1}^N (m_{h_a}^2 |h_a^+|^2 + m_{k_a}^2 |k_a^{++}|^2) - m_S^2 (S^2 + \text{c.c.}) - m_S'^2 |S|^2 - \sum_{a,b,c=1}^N \mu_{abc} (h_a^- h_b^- k_c^{++} + \text{c.c.}) \\ & + \lambda_\Phi |\Phi|^4 + \sum_{a,b,c,d=1}^N (\lambda_{h_{abcd}} h_a^+ h_b^+ h_c^- h_d^- + \lambda_{k_{abcd}} k_a^{++} k_b^{++} k_c^{--} k_d^{--}) + \lambda_{h_{ab} k_{cd}} h_a^+ h_b^- k_c^{++} k_d^{--} \\ & + \lambda_S (S^4 + \text{c.c.}) + \lambda'_S |S|^4 + \lambda''_S |S|^2 (S^2 + \text{c.c.}) + \lambda_{\Phi S} |\Phi|^2 (S^2 + \text{c.c.}) + \lambda'_{\Phi S} |\Phi|^2 |S|^2 \\ & + \sum_{a,b=1}^N (\lambda_{\Phi h_{ab}} |\Phi|^2 h_a^+ h_b^- + \lambda_{\Phi k_{ab}} |\Phi|^2 k_a^{++} k_b^{--} + \lambda_{h_{ab} S} h_a^+ h_b^- (S^2 + \text{c.c.}) + \lambda'_{h_{ab} S} h_a^+ h_b^- |S|^2 \\ & + \lambda_{k_{ab} S} k_a^{++} k_b^{--} (S^2 + \text{c.c.}) + \lambda'_{k_{ab} S} k_a^{++} k_b^{--} |S|^2), \end{aligned} \quad (\text{II.2})$$

where we define $S = (S_R + iS_I)/\sqrt{2}$, τ_2 is a second component of the Pauli matrix, the index $i(j)$ runs 1-3, and M_E can be diagonal without loss of the generality. Notice here that g should be symmetric matrix. We work on the basis where all the coefficients are real and positive for our brevity. We parametrize these scalar fields as

$$\Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}, \quad \phi^0 = \frac{1}{\sqrt{2}}(v + h + ia), \quad (\text{II.3})$$

where h is the SM-like Higgs, and v is related to the Fermi constant G_F by $v^2 = 1/(\sqrt{2}G_F) \approx (246 \text{ GeV})^2$. The VEV of Φ is derived from the tadpole condition $\partial\mathcal{V}/\partial v = 0$ such that $v^2 \simeq \frac{m_\Phi^2}{\lambda_\Phi}$. After the electroweak symmetry breaking, we have massive gauge bosons W^\pm and Z by which the NG bosons ϕ^\pm and a are absorbed respectively. Each of scalar boson mass eigenvalue is straightforwardly obtained by the potential where we adopt these masses as a free parameters in our analysis below. The charged lepton sector is the same as the SM lepton, that is, the charged lepton mass is given by the term of y_ℓ after the electroweak symmetry breaking.

The exotic vector-like $\text{SU}(2)_L$ doublet L' is given as follow:

$$L'_{L(R)} \equiv \begin{bmatrix} N \\ E^- \end{bmatrix}_{L(R)}. \quad (\text{II.4})$$

The mass term of L' is given by $M_L(\bar{E}_L^- E_R^- + \bar{N}_L N_R)$ at the tree level. The mass difference of E^- and N is induced at the one-loop level via gauge interaction where the mass of E^- becomes heavier by $O(300)$ MeV for $M_L \sim 1$ TeV [108, 109]. For neutral component, we have Majorana fermions N_L and N_R^c . Then its mass matrix in the basis of $[N_L, N_R^c]$ is given by

$$M_N = \begin{bmatrix} \delta m & M_L \\ M_L & 0 \end{bmatrix}, \quad (\text{II.5})$$

where $\delta m(<< M_L)$ is induced at the Zee-Babu type of two-loop level [3], and we assume to be one generation case with positive real for our simple analysis. Here the loop function is the same as the Zee-Babu model. M_N is diagonalized by 2×2 unitary mixing matrix V_N as $V_N M_N V_N^T \approx \text{diag.}(M_L - \delta m/2, M_L + \delta m/2) \equiv \text{diag.}(M_{N_1}, M_{N_2})$, where we assume that V_N is approximately a maximal mixing as follow:

$$V_N \approx \frac{1}{\sqrt{2}} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}, \quad (\text{II.6})$$

then one finds our Majorana fields ψ_1 and ψ_2 , by redefining $\psi_1 \equiv N_1 + N_1^c$ and $\psi_2 \equiv N_2^c + N_2$ [87]. The lighter field ψ_1 can be a stable DM candidate and ψ_2 can be a meta-stable DM candidate, but the mass difference between them is tiny because δm is generated at the two loop level. Thus we take account of the co-annihilation system including ψ_1 , ψ_2 and E^- to obtain the relic density.

III. PHENOMENOLOGY OF THE MODEL

In this section, we carry out phenomenological study of our model such as active neutrino mass, lepton flavor violation and dark matter, based on the set up discussed in the previous section.

A. Active neutrino mass

The active neutrino mass matrix is generated at the three-loop level, but we compute the mass insertion method. Therefore the mass matrix is effectively induced at the one-loop level as follows:

$$(m_\nu)_{ab} = - \sum_{j=1}^3 \frac{(f_R)_{aj} \delta m_j (f_R)_{bj} (m_{S_R}^2 - m_{S_I}^2) M_{L_j}^2}{(4\pi)^2 M_{\max}^4} F_1(X_{L_j}, X_{S_R}, X_{S_I}), \quad (\text{III.1})$$

$$F_1(X_{L_j}, X_{S_R}, X_{S_I}) \equiv \int dx dy dz \frac{x \delta(x+y+z-1)}{(xX_{L_j} + yX_{S_R} + zX_{S_I})^2}, \quad (\text{III.2})$$

where we define $X_f \equiv \left(\frac{m_f}{M_{\max}}\right)^2$ and $M_{\max} \equiv \text{Max}[M_{L_j}, m_{S_R}, m_{S_I}]$.

Then $(m_\nu)_{ab}$ is diagonalized by the Maki-Nakagawa-Sakata mixing matrix V_{MNS} (MNS) as

$$(m_\nu)_{ab} = (V_{\text{MNS}}^* D_\nu V_{\text{MNS}}^\dagger)_{ab}, \quad D_\nu \equiv (m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \quad (\text{III.3})$$

$$V_{\text{MNS}} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \quad (\text{III.4})$$

where we neglect the Majorana phase. The following neutrino oscillation data at 95% confidence level [110] is given as

$$0.2911 \leq s_{12}^2 \leq 0.3161, \quad 0.5262 \leq s_{23}^2 \leq 0.5485, \quad 0.0223 \leq s_{13}^2 \leq 0.0246, \quad (\text{III.5})$$

$$|m_{\nu_3}^2 - m_{\nu_2}^2| = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2, \quad m_{\nu_2}^2 - m_{\nu_1}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2,$$

where we assume one of three neutrino masses is zero with normal ordering in our analysis below.

To achieve the numerical analysis, we apply the Casas-Ibarra parametrization [111] to our form. Thus the Yukawa coupling f_R is rewritten as

$$f_R = V_{\text{MNS}}^* \sqrt{D_\nu} O \sqrt{R}, \quad (\text{III.6})$$

Process	(i, j)	Experimental bounds (90% CL)
$\mu^- \rightarrow e^- \gamma$	(2, 1)	$\text{Br}(\mu \rightarrow e \gamma) < 5.7 \times 10^{-13}$
$\tau^- \rightarrow e^- \gamma$	(3, 1)	$\text{Br}(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}$
$\tau^- \rightarrow \mu^- \gamma$	(3, 2)	$\text{Br}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$

TABLE III: Summary of $\ell_i \rightarrow \ell_j \gamma$ process and the lower bound of experimental data [?].

where O , which is an complex orthogonal matrix, and R , which is a diagonal matrix, are respectively formulated as

$$O = \begin{pmatrix} 0 & 0 & 1 \\ \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \end{pmatrix}, \quad \alpha \text{ is a complex parameter}, \quad (\text{III.7})$$

and

$$R_{jj} = \frac{\delta m_j (m_{S_R}^2 - m_{S_I}^2) M_{L_j}^2}{(4\pi)^2 M_{\max}^4} F_1(X_{L_j}, X_{S_R}, X_{S_I}). \quad (\text{III.8})$$

Notice here that we assume the lightest neutrino mass is zero and the neutrino mass spectrum is normal hierarchy without Majorana and Dirac phase in the numerical analysis below for brevity. The simplification of the mixing matrix O is the direct consequence of its massless neutrino for the first generation.

Lepton flavor violation processes: We have the lepton flavor violations such as $\ell_i \rightarrow \ell_j \gamma$,¹ and each of flavor dependent process has to satisfy the current upper bound, as can be seen in Table III. Notice here that $\mu \rightarrow e \gamma$ process gives the most stringent upper bound. Our branching fraction is given by

$$\text{Br}(\ell_i \rightarrow \ell_j \gamma) = \frac{3\alpha_{\text{em}}}{64\pi G_F^2} |G_{ij}|^2, \quad (\text{III.9})$$

$$G_{ij} \equiv \sum_{k=1-3} (f_R)_{i,k} (f_R^\dagger)_{k,j} \left[F_2 \left(\frac{m_{S_R}^2}{M_{L_k}^2} \right) + F_2 \left(\frac{m_{S_I}^2}{M_{L_k}^2} \right) \right], \quad (\text{III.10})$$

$$F_2(x) \equiv \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4}, \quad (\text{III.11})$$

where $G_F \approx 1.1 \times 10^{-5} [\text{GeV}^{-2}]$ is Fermi constant, and $\alpha_{\text{em}} \approx 1/137$ is the fine-structure constant.

¹ However there exist $\ell_i^- \rightarrow \ell_j^- \ell_k^+ \ell_\ell^-$ processes at the one-loop level, we neglect them since they are subdominant processes [112].

Numerical results: Now we discuss the numerical solutions to satisfy the neutrino oscillation data and the LFV constraints. Here we randomly select the values of these parameters within the following ranges:

$$\begin{aligned} M_{L_1} &= (100 \text{ GeV}, 2000 \text{ GeV}), M_{L_{2,3}} = (M_{L_1}, 2000 \text{ GeV}), m_{S_R} = (M_{L_1}, 2000 \text{ GeV}), \\ m_{S_I} &= (M_{L_1}, 2000 \text{ GeV}), \delta m = (200 \text{ keV}, 400 \text{ keV}), \end{aligned} \quad (\text{III.12})$$

where all the elements of f_R is taken to be $f_R < \sqrt{4\pi}$ as the perturbative limit, and 10^5 sampling points are our examinations to search for our allowed parameters. We find that 386 solutions over all the range that we take, and all the typical absolute order of f_R is $\mathcal{O}(0.01)$.

B. Estimation of the scale δm

In this section we estimate the scale of δm that is generated at the two-loop level through the Zee-Babu like diagram, where we assume that all the off-diagonal elements for related Yukawa couplings (f_L and g) are zero. Thus we can evade any kinds of LFV processes, even when we take rather larger values of diagonal Yukawa couplings. Then the mass formula is given by

$$\delta m \approx \frac{8N^3 \mu f_L^2 g m_\tau^2}{(4\pi)^4 m_{k^{\pm\pm}}^2} F_3(X), \quad (\text{III.13})$$

$$F_3(X) \equiv \int_0^1 dx \int_0^1 d\alpha \int_0^{1-y} dy \int_0^{1-\alpha} d\beta [(y^2 - y)\alpha X - \beta(xX + y)]^{-1}, \quad (\text{III.14})$$

where $X \equiv (m_{h^\pm}/m_{k^{\pm\pm}})^2$, $m_\tau (= 1.776 \text{ GeV})$ is the tau lepton mass, and neglect the masses of the charged leptons in the loop function $F_3(X)$. We assume $m_{h_1} = m_{h_2} = \dots m_h, m_{k_1} = m_{k_2} = \dots m_k, f_L^1 = f_L^2 = \dots f_L$ and $g^1 = g^2 = \dots g$. Then it simplifies $F_3(X) \approx 1/X$. *The important constraint comes from the inelastic direct detection searches via Z boson portal that suggests that $\mathcal{O}(200 \text{ keV}) \lesssim \delta m$.* Also considering the correction bound for the charged boson masses, the trilinear coupling μ can roughly be restricted as $\mu \lesssim (4\pi) \text{Min}[m_h^\pm, m_k^{\pm\pm}]$ [21]. Here we assume $m_k^{\pm\pm} \ll m_h^\pm$ for our convenience and adopt the upper limit of the trilinear coupling μ in the following analysis. Moreover, we hereafter fix the mass of $k^{\pm\pm}$ to be $m_k^{\pm\pm} \approx 700 \text{ GeV}$ which is sufficiently larger than the lowest bound by doubly charged scalar search at the LHC [113]. Applying these requirements to the mass formula, we obtain the following relation

$$\frac{0.05X m_{k^{\pm\pm}} [\text{GeV}]}{m_\tau^2} \lesssim (NY)^3 \rightarrow \left[\frac{0.05X m_{k^{\pm\pm}} [\text{GeV}]}{m_\tau^2} \right]^{1/3} \lesssim NY, \quad (\text{III.15})$$

where we assume $Y \equiv f_L = g$ for brevity. In this case we have to be careful of the fact that the N negatively enlarges the $(g-2)_\mu$, although the LFVs are still zero as far as the diagonal

$Y(= f_L = g)$. We thus consider the upper limit for the absolute value of $(g - 2)_\mu$ which is given by

$$|\Delta a_\mu| \approx \left| -\frac{|NY|^2}{3(4\pi)^2} \frac{m_\mu^2}{m_{k^{\pm\pm}}^2} \right| \lesssim 40 \times 10^{-10}, \quad (\text{III.16})$$

where the last inequality represents the absolute upper bound from the experimental result in ref. [114]. Combining with Eq. (III.15) and $m_k^{\pm\pm} \approx 700$ GeV, we obtain the following upper mass bound for the singly charged bosons h^\pm

$$m_{h^\pm} \lesssim 6 \text{ TeV}. \quad (\text{III.17})$$

The upper bound plays an role in determining the lower mass bound of h^\pm in analyzing the DM relic density, as will be discussed below.

C. Dark Matter

We consider fermionic DM candidates $X(\equiv \psi_1)$, which is assumed to be the lightest and stable particle, and ψ_2 , which is assumed to be the meta-stable particle. Here we analyze the one flavor case for simplicity. Since the mass difference between ψ_1 and ψ_2 is tiny due to generating the two-loop level, we have to consider the measured relic density on the co-annihilation system. Furthermore, the mass of E^- is also close to that of DM. Thus we assume to be $M_X \approx M_{\psi_2} \approx M_E$ throughout the DM analysis.

Relic density: The relic density of DM is obtained by calculating annihilation and coannihilation cross sections where our dominant contribution comes from the kinetic term of L' and the term with f_L . For gauge interactions, the complete analysis is provided in ref. [109]. In addition to gauge interactions, the (co-)annihilation cross section from the Yukawa interactions is given by

$$\sigma v_{\text{rel}} \approx \frac{|NY|^4 r^2 (1 - 2r + 2r^2)}{48\pi M_X^2} v_{\text{rel}}^2, \quad (\text{III.18})$$

where $r \equiv M_X^2/(m_{h^\pm}^2 + M_X^2)$. Notice here that we eliminate the NY dependence by substituting Eq. (III.15). Adding the above p-wave to the effective whole p-wave, we estimate the relic density in terms of the DM mass as can be seen in Fig. 1, applying the approximated formulae in refs. [87, 115]. In this figure, each of the colored line; (blue, green, red), represents $m_{h^\pm} = (6, 4, 2)$ TeV, and the black horizontal is the observed relic density; $\Omega h^2 \approx 0.12$ [116]. Notice here that $m_{h^\pm} = 6$ TeV comes from the upper constraint for the $(g - 2)_\mu$ discussed in Eq. (III.17).

In case where $m_{h^\pm} = 6$ TeV, the Yukawa contribution is negligible due to the suppression of the heavy mass of h^\pm , and the allowed DM mass is about 1.12 TeV, which is consistent of the

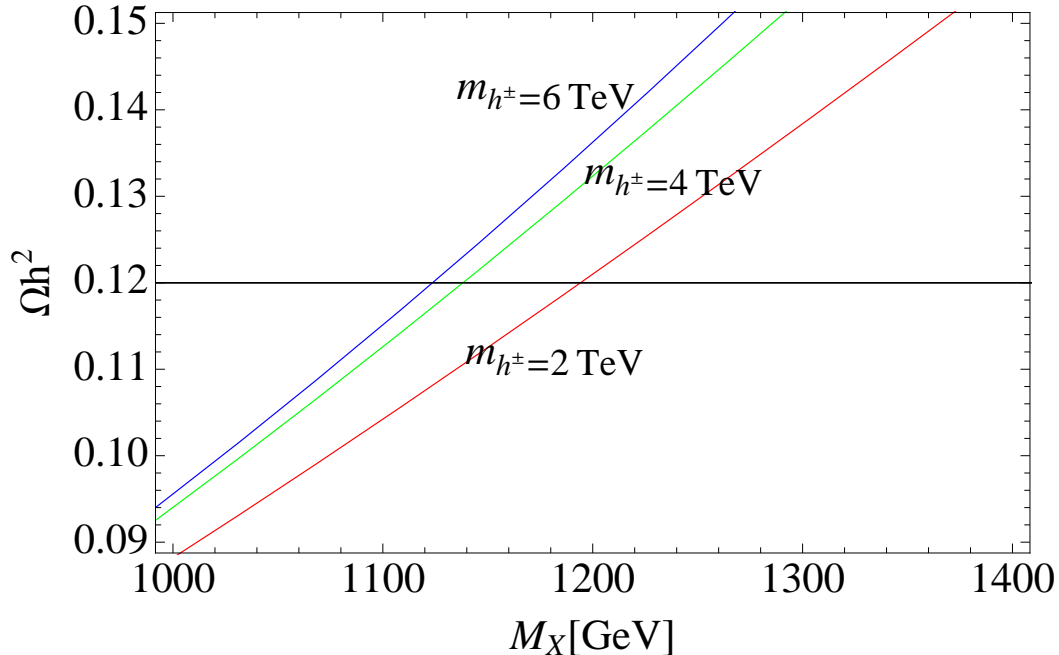


FIG. 1: The relic density in terms of the DM mass. Each of the blue, green, and red line corresponds to $m_{h^\pm} = (6, 4, 2)$ TeV. The black horizontal is the observed relic density; $\Omega h^2 \approx 0.12$.

result to the ref [109].² It suggests that the lower mass of the singly charged boson is 1.12 TeV to kinematically forbid the too rapid two body decay $X \rightarrow h^\pm \ell$ through the term f_L . Therefore, we obtain the following relation

$$1.12 \text{ TeV} \lesssim m_{h^\pm} \lesssim 6 \text{ TeV}. \quad (\text{III.19})$$

In case where $m_{h^\pm} = (4, 2)$ TeV, on the other hand, the contribution from Yukawa interactions becomes to be relevant in relic density estimation. Thus the value of relic density decrease at the same DM mass since the (co-)annihilation cross sections become larger. As a result, we obtain the allowed DM mass at (1.14, 1.2) TeV, respectively.

Photon excess: It might be worth mentioning the photon excess such that the emitting photon energy E_γ can be replaced by the mass difference (δm) between ψ_1 and ψ_2 . Since our model has the lower bound; $\mathcal{O}(200) \text{ keV} \lesssim E_\gamma$ to evade the constraint from direct detection searches with spin independent inelastic scattering process via Z boson portal, the X-ray line at the 3.55 keV energy cannot be explained. However, we still expect that the other experiments might detect our energy scale near future. As one of the examples, we focus on the 511 keV photon excess by

² In this reference, they consider the gauge interactions only.

INTEGRAL [117] and briefly discuss. The INTEGRAL experiment provides the gamma ray line flux at around 511 keV as

$$\Phi_\gamma(511\text{keV}) = (1.05 \pm 0.06) \times 10^{-3} \text{ photon cm}^{-2}\text{s}^{-1}. \quad (\text{III.20})$$

On the other hand, our gamma ray flux is found as [118]

$$\Phi_\gamma(511\text{keV}) \approx \frac{10^{27}\text{s}}{2} \left[\frac{\Gamma(X_2 \rightarrow X\gamma)}{M_X(\text{MeV}^{-1})} \right] \times 10^{-3} \text{ photon cm}^{-2}\text{s}^{-1}, \quad (\text{III.21})$$

where

$$\frac{\Gamma(X_2 \rightarrow X\gamma)}{M_X} \approx \frac{\alpha_{\text{em}} \delta m^3 M_X}{64\pi^4 m_{h^\pm}^4} \left| \sum_k^{1-3} \text{Im}[(f_R)_{k1}(f_R)_{k2}^*] G\left(\frac{M_X^2}{m_{h^\pm}^2}\right) \right|^2, \quad (\text{III.22})$$

$$G(X) \equiv \int_0^1 \frac{dx x(x-1)}{x^2 X - (1+X)x + 1}. \quad (\text{III.23})$$

Comparing with our formula and the experimental result, we obtain the following constraint for the Yukawa couplings f_R

$$1.9 \times 10^{-10} \lesssim \left| \sum_k^{1-3} \text{Im}[(f_R)_{k1}(f_R)_{k2}^*] \right| \lesssim 2.0 \times 10^{-10} \text{ for } m_{h^\pm} = 6 \text{ TeV} (M_X = 1.12 \text{ TeV}), \quad (\text{III.24})$$

$$8.2 \times 10^{-11} \lesssim \left| \sum_k^{1-3} \text{Im}[(f_R)_{k1}(f_R)_{k2}^*] \right| \lesssim 8.7 \times 10^{-10} \text{ for } m_{h^\pm} = 4 \text{ TeV} (M_X = 1.14 \text{ TeV}), \quad (\text{III.25})$$

$$1.6 \times 10^{-11} \lesssim \left| \sum_k^{1-3} \text{Im}[(f_R)_{k1}(f_R)_{k2}^*] \right| \lesssim 1.7 \times 10^{-11} \text{ for } m_{h^\pm} = 2 \text{ TeV} (M_X = 1.2 \text{ TeV}), \quad (\text{III.26})$$

where we have used each the solution for (m_{h^\pm}, M_X) in analyzing the relic density. It suggests that the Yukawa CP phases are very small, and this can easily be realized, satisfying the neutrino oscillation data.

IV. CONCLUSIONS AND DISCUSSIONS

We have studied radiatively induced neutrino mass model, in which we have used the mass insertion method to the neutrino sector. As a result, neutrino mass is effectively generated at the one-loop level with the degenerated neutral fermions masses which obtain mass splitting at the two loop level. Considering the constraints of LFV processes, we have numerically analyzed the allowed region search to satisfy the LFVs and the neutrino oscillation data. Then we have discussed the DM phenomenologies, in which we have firstly investigated the constraint of inelastic scattering direct detection searches through the SM neutral vector boson portal. The experiment suggests that the

mass difference between the degenerated masses should be larger than the order \mathcal{O} (200 keV). We have found the solution to evade this constraint by introducing the multiple charged bosons with the same quantum number as these fields that we originally be introduced. This modification is in favor of the recent experimental result of the diphoton excess at the 750 GeV reported by CMS [119] and ATLAS [120] where multiple charged scalars enhance diphoton decay branching ratio of 750 GeV neutral scalar interacting with these charged scalars [121–131]. However it simultaneously induces the negative enhancement of the muon anomalous magnetic moment. Considering the upper absolute value $|\Delta a_\mu| \lesssim 40 \times 10^{-10}$, we have obtained the upper bound for the singly charged boson mass; $m_{h^\pm} \lesssim 6$ TeV. Applying this upper bound, we have analyzed the DM relic density with co-annihilation system and found that the dominant annihilation mode comes from the kinetic term of L' in case of $m_{h^\pm} = 6$ TeV. But if the mass of h^\pm decreases, the value of relic density also decreases at the same DM mass, therefore the allowed DM mass increases in fig. 1. Then we have briefly discussed the possibility to detect photon emission through indirect detection searches larger than our threshold energy $\simeq \mathcal{O}$ (200 keV). Here we have focused on the INTEGRAL experiment at 511 keV photon energy, and we have found that the imaginary part of Yukawa coupling f_R is constrained as $|\sum_k^{1-3} \text{Im}[(f_R)_{k1}(f_R)_{k2}^*]| \approx \mathcal{O}(10^{-10} - 10^{-11})$, depending on our three bench mark sets of (m_{h^\pm}, M_X) obtained by the analysis of relic density.

It might be worth mentioning the muon anomalous magnetic moment in our model. We have new sources of positive contribution from f_R and negative contribution from f_L , but the positive contribution is constrained by the LFVs, therefore f_R is typically the order $\mathcal{O}(0.01)$ as discussed before. Hence our total $(g - 2)_\mu$ tends to be negative value, which is against the experimental result. Thus we have used this result as the upper bound to restrict the charged boson. If future experiments could provide a negative contribution, it could still be verifiable.

The extra charged particles could be produced at the LHC since the masses of these particles are around $\mathcal{O}(1)$ TeV in our model. Thus we can test our model by future experiments searching for exotic charged particles. The detailed analysis is beyond the scope of our study and will be given in future work.

Acknowledgments

Authors thank to Dr. Kei Yagyu for fruitful discussions. H.O. expresses his sincere gratitude toward all the KIAS members, Korean cordial persons, foods, culture, weather, and all the other

things. This work was supported by the Korea Neutrino Research Center which is established by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIP) (No. 2009-0083526) (Y.O.).

-
- [1] A. Zee, Phys. Lett. B **93**, 389 (1980) [Erratum-ibid. B **95**, 461 (1980)].
 - [2] T. P. Cheng and L. F. Li, Phys. Rev. D **22**, 2860 (1980).
 - [3] A. Zee, Nucl. Phys. B **264**, 99 (1986); K. S. Babu, Phys. Lett. B **203**, 132 (1988).
 - [4] L. M. Krauss, S. Nasri and M. Trodden, Phys. Rev. D **67**, 085002 (2003) [arXiv:hep-ph/0210389].
 - [5] E. Ma, Phys. Rev. D **73**, 077301 (2006) [hep-ph/0601225].
 - [6] M. Aoki, S. Kanemura and O. Seto, Phys. Rev. Lett. **102**, 051805 (2009) [arXiv:0807.0361].
 - [7] M. Gustafsson, J. M. No and M. A. Rivera, Phys. Rev. Lett. **110**, 211802 (2013) arXiv:1212.4806 [hep-ph].
 - [8] T. Hambye, K. Kannike, E. Ma and M. Raidal, Phys. Rev. D **75**, 095003 (2007) [hep-ph/0609228].
 - [9] P. -H. Gu and U. Sarkar, Phys. Rev. D **77**, 105031 (2008) [arXiv:0712.2933 [hep-ph]].
 - [10] N. Sahu and U. Sarkar, Phys. Rev. D **78**, 115013 (2008) [arXiv:0804.2072 [hep-ph]].
 - [11] P. -H. Gu and U. Sarkar, Phys. Rev. D **78**, 073012 (2008) [arXiv:0807.0270 [hep-ph]].
 - [12] K. S. Babu and C. Macesanu, Phys. Rev. D **67**, 073010 (2003) [hep-ph/0212058].
 - [13] D. Aristizabal Sierra and D. Restrepo, JHEP **0608**, 036 (2006) [hep-ph/0604012].
 - [14] D. Aristizabal Sierra and M. Hirsch, JHEP **0612**, 052 (2006) [hep-ph/0609307].
 - [15] M. Nebot, J. F. Oliver, D. Palao and A. Santamaria, Phys. Rev. D **77**, 093013 (2008) [arXiv:0711.0483 [hep-ph]].
 - [16] R. Bouchand and A. Merle, JHEP **1207**, 084 (2012) [arXiv:1205.0008 [hep-ph]].
 - [17] Y. Kajiyama, H. Okada and K. Yagyu, Nucl. Phys. B **887**, 358 (2014) [arXiv:1309.6234 [hep-ph]].
 - [18] K. L. McDonald, JHEP **1311**, 131 (2013) [arXiv:1310.0609 [hep-ph]].
 - [19] E. Ma, Phys. Lett. B **732**, 167 (2014) [arXiv:1401.3284 [hep-ph]].
 - [20] D. Schmidt, T. Schwetz and H. Zhang, Nucl. Phys. B **885**, 524 (2014) [arXiv:1402.2251 [hep-ph]].
 - [21] J. Herrero-Garcia, M. Nebot, N. Rius and A. Santamaria, Nucl. Phys. B **885**, 542 (2014) [arXiv:1402.4491 [hep-ph]].
 - [22] A. Ahriche, S. Nasri and R. Soualah, Phys. Rev. D **89**, 095010 (2014) [arXiv:1403.5694 [hep-ph]].
 - [23] H. N. Long and V. V. Vien, Int. J. Mod. Phys. A **29**, no. 13, 1450072 (2014) [arXiv:1405.1622

[hep-ph]].

- [24] V. Van Vien, H. N. Long and P. N. Thu, arXiv:1407.8286 [hep-ph].
- [25] M. Aoki, S. Kanemura, T. Shindou and K. Yagyu, JHEP **1007**, 084 (2010) [Erratum-ibid. **1011**, 049 (2010)] [arXiv:1005.5159 [hep-ph]].
- [26] S. Kanemura, O. Seto and T. Shimomura, Phys. Rev. D **84**, 016004 (2011) [arXiv:1101.5713 [hep-ph]].
- [27] M. Lindner, D. Schmidt and T. Schwetz, Phys. Lett. B **705**, 324 (2011) [arXiv:1105.4626 [hep-ph]].
- [28] S. Kanemura, T. Nabeshima and H. Sugiyama, Phys. Lett. B **703**, 66 (2011) [arXiv:1106.2480 [hep-ph]].
- [29] M. Aoki, J. Kubo, T. Okawa and H. Takano, Phys. Lett. B **707**, 107 (2012) [arXiv:1110.5403 [hep-ph]].
- [30] S. Kanemura, T. Nabeshima and H. Sugiyama, Phys. Rev. D **85**, 033004 (2012) [arXiv:1111.0599 [hep-ph]].
- [31] D. Schmidt, T. Schwetz and T. Toma, Phys. Rev. D **85**, 073009 (2012) [arXiv:1201.0906 [hep-ph]].
- [32] S. Kanemura and H. Sugiyama, Phys. Rev. D **86**, 073006 (2012) [arXiv:1202.5231 [hep-ph]].
- [33] Y. Farzan and E. Ma, Phys. Rev. D **86**, 033007 (2012) [arXiv:1204.4890 [hep-ph]].
- [34] K. Kumericki, I. Picek and B. Radovic, JHEP **1207**, 039 (2012) [arXiv:1204.6597 [hep-ph]].
- [35] K. Kumericki, I. Picek and B. Radovic, Phys. Rev. D **86**, 013006 (2012) [arXiv:1204.6599 [hep-ph]].
- [36] E. Ma, Phys. Lett. B **717**, 235 (2012) [arXiv:1206.1812 [hep-ph]].
- [37] G. Gil, P. Chankowski and M. Krawczyk, Phys. Lett. B **717**, 396 (2012) [arXiv:1207.0084 [hep-ph]].
- [38] H. Okada and T. Toma, Phys. Rev. D **86**, 033011 (2012) arXiv:1207.0864 [hep-ph].
- [39] D. Hehn and A. Ibarra, Phys. Lett. B **718**, 988 (2013) [arXiv:1208.3162 [hep-ph]].
- [40] S. Baek, P. Ko, H. Okada and E. Senaha, JHEP **1409**, 153 (2014) [arXiv:1209.1685 [hep-ph]].
- [41] P. S. B. Dev and A. Pilaftsis, Phys. Rev. D **86**, 113001 (2012) [arXiv:1209.4051 [hep-ph]].
- [42] Y. Kajiyama, H. Okada and T. Toma, Eur. Phys. J. C **73**, 2381 (2013) [arXiv:1210.2305 [hep-ph]].
- [43] M. Kohda, H. Sugiyama and K. Tsumura, Phys. Lett. B **718**, 1436 (2013) [arXiv:1210.5622 [hep-ph]].
- [44] M. Aoki, J. Kubo and H. Takano, Phys. Rev. D **87**, no. 11, 116001 (2013) [arXiv:1302.3936 [hep-ph]].
- [45] Y. Kajiyama, H. Okada and K. Yagyu, Nucl. Phys. B **874**, 198 (2013) [arXiv:1303.3463 [hep-ph]].
- [46] Y. Kajiyama, H. Okada and T. Toma, Phys. Rev. D **88**, 015029 (2013) [arXiv:1303.7356].

- [47] S. Kanemura, T. Matsui and H. Sugiyama, Phys. Lett. B **727**, 151 (2013) [arXiv:1305.4521 [hep-ph]].
- [48] S. S. C. Law and K. L. McDonald, JHEP **1309**, 092 (2013) [arXiv:1305.6467 [hep-ph]].
- [49] B. Dasgupta, E. Ma and K. Tsumura, Phys. Rev. D **89**, 041702 (2014) [arXiv:1308.4138 [hep-ph]].
- [50] S. Baek, H. Okada and T. Toma, JCAP **1406**, 027 (2014) [arXiv:1312.3761 [hep-ph]].
- [51] H. Okada, arXiv:1404.0280 [hep-ph].
- [52] A. Ahriche, C. S. Chen, K. L. McDonald and S. Nasri, Phys. Rev. D **90**, no. 1, 015024 (2014) [arXiv:1404.2696 [hep-ph]].
- [53] A. Ahriche, K. L. McDonald and S. Nasri, JHEP **1410**, 167 (2014) [arXiv:1404.5917 [hep-ph]].
- [54] C. -S. Chen, K. L. McDonald and S. Nasri, Phys. Lett. B **734**, 388 (2014) [arXiv:1404.6033 [hep-ph]].
- [55] S. Kanemura, T. Matsui and H. Sugiyama, Phys. Rev. D **90**, 013001 (2014) [arXiv:1405.1935 [hep-ph]].
- [56] H. Okada and Y. Orikasa, Phys. Rev. D **90**, no. 7, 075023 (2014) [arXiv:1407.2543 [hep-ph]].
- [57] S. Fraser, E. Ma and O. Popov, Phys. Lett. B **737**, 280 (2014) [arXiv:1408.4785 [hep-ph]].
- [58] H. Okada, T. Toma and K. Yagyu, Phys. Rev. D **90**, no. 9, 095005 (2014) [arXiv:1408.0961 [hep-ph]].
- [59] H. Hatanaka, K. Nishiwaki, H. Okada and Y. Orikasa, Nucl. Phys. B **894**, 268 (2015) [arXiv:1412.8664 [hep-ph]].
- [60] S. Baek, H. Okada and K. Yagyu, JHEP **1504**, 049 (2015) [arXiv:1501.01530 [hep-ph]].
- [61] L. G. Jin, R. Tang and F. Zhang, Phys. Lett. B **741**, 163 (2015) [arXiv:1501.02020 [hep-ph]].
- [62] P. Culjak, K. Kumericki and I. Picek, Phys. Lett. B **744**, 237 (2015) [arXiv:1502.07887 [hep-ph]].
- [63] H. Okada, arXiv:1503.04557 [hep-ph].
- [64] C. Q. Geng and L. H. Tsai, arXiv:1503.06987 [hep-ph].
- [65] H. Okada, N. Okada and Y. Orikasa, arXiv:1504.01204 [hep-ph].
- [66] C. Q. Geng, D. Huang and L. H. Tsai, Phys. Lett. B **745**, 56 (2015) [arXiv:1504.05468 [hep-ph]].
- [67] A. Ahriche, K. L. McDonald, S. Nasri and T. Toma, Phys. Lett. B **746**, 430 (2015) [arXiv:1504.05755 [hep-ph]].
- [68] D. Restrepo, A. Rivera, M. Sánchez-Peláez, O. Zapata and W. Tangarife, arXiv:1504.07892 [hep-ph].
- [69] S. Kashiwase, H. Okada, Y. Orikasa and T. Toma, arXiv:1505.04665 [hep-ph].

- [70] K. Nishiwaki, H. Okada and Y. Orikasa, arXiv:1507.02412 [hep-ph].
- [71] W. Wang and Z. L. Han, Phys. Rev. D **92**, 095001 (2015) [arXiv:1508.00706 [hep-ph]].
- [72] H. Okada and K. Yagyu, arXiv:1508.01046 [hep-ph].
- [73] A. Ahriche, K. L. McDonald and S. Nasri, arXiv:1508.02607 [hep-ph].
- [74] Y. H. Ahn and H. Okada, Phys. Rev. D **85**, 073010 (2012) [arXiv:1201.4436 [hep-ph]].
- [75] E. Ma, A. Natale and A. Rashed, Int. J. Mod. Phys. A **27**, 1250134 (2012) [arXiv:1206.1570 [hep-ph]].
- [76] Y. Kajiyama, H. Okada and K. Yagyu, JHEP **10**, 196 (2013) arXiv:1307.0480 [hep-ph].
- [77] A. E. Carcamo Hernandez, I. d. M. Varzielas, S. G. Kovalenko, H. Päs and I. Schmidt, Phys. Rev. D **88**, 076014 (2013) [arXiv:1307.6499 [hep-ph]].
- [78] E. Ma and A. Natale, Phys. Lett. B **723**, 403 (2014) [arXiv:1403.6772 [hep-ph]].
- [79] M. Aoki and T. Toma, JCAP **1409**, 016 (2014) [arXiv:1405.5870 [hep-ph]].
- [80] E. Ma, Phys. Lett. B **741**, 202 (2015) [arXiv:1411.6679 [hep-ph]].
- [81] E. Ma, arXiv:1504.02086 [hep-ph].
- [82] E. Ma, Phys. Rev. Lett. **112**, 091801 (2014) [arXiv:1311.3213 [hep-ph]].
- [83] H. Okada and K. Yagyu, Phys. Rev. D **89**, 053008 (2014) [arXiv:1311.4360 [hep-ph]].
- [84] S. Baek, H. Okada and T. Toma, Phys. Lett. B **732**, 85 (2014) [arXiv:1401.6921 [hep-ph]].
- [85] H. Okada and K. Yagyu, Phys. Rev. D **90**, no. 3, 035019 (2014) [arXiv:1405.2368 [hep-ph]].
- [86] V. Brdar, I. Picek and B. Radovcic, Phys. Lett. B **728**, 198 (2014) [arXiv:1310.3183 [hep-ph]].
- [87] H. Okada and Y. Orikasa, arXiv:1509.04068 [hep-ph].
- [88] H. Okada, Y. Orikasa and T. Toma, arXiv:1511.01018 [hep-ph].
- [89] S. Fraser, C. Kownacki, E. Ma and O. Popov, arXiv:1511.06375 [hep-ph].
- [90] S. Fraser, E. Ma and M. Zakeri, arXiv:1511.07458 [hep-ph].
- [91] R. Adhikari, D. Borah and E. Ma, arXiv:1512.05491 [hep-ph].
- [92] S. Kanemura and H. Sugiyama, Phys. Lett. B **753**, 161 (2016) [arXiv:1510.08726 [hep-ph]].
- [93] F. Bonnet, M. Hirsch, T. Ota and W. Winter, JHEP **1207**, 153 (2012) [arXiv:1204.5862 [hep-ph]].
- [94] D. Aristizabal Sierra, A. Degee, L. Dorame and M. Hirsch, JHEP **1503**, 040 (2015) [arXiv:1411.7038 [hep-ph]].
- [95] H. Davoudiasl and I. M. Lewis, Phys. Rev. D **90**, no. 3, 033003 (2014) [arXiv:1404.6260 [hep-ph]].
- [96] M. Lindner, S. Schmidt and J. Smirnov, arXiv:1405.6204 [hep-ph].
- [97] H. Okada and Y. Orikasa, arXiv:1412.3616 [hep-ph].

- [98] J. March-Russell, C. McCabe and M. McCullough, JHEP **1003**, 108 (2010) [arXiv:0911.4489 [hep-ph]].
- [99] S. F. King, A. Merle and L. Panizzi, arXiv:1406.4137 [hep-ph].
- [100] Y. Mambrini, S. Profumo and F. S. Queiroz, arXiv:1508.06635 [hep-ph].
- [101] S. M. Boucenna, S. Morisi and J. W. F. Valle, Adv. High Energy Phys. **2014**, 831598 (2014) [arXiv:1404.3751 [hep-ph]].
- [102] A. Ahriche, S. M. Boucenna and S. Nasri, arXiv:1601.04336 [hep-ph].
- [103] H. Okada and Y. Orikasa, arXiv:1512.06687 [hep-ph].
- [104] A. Ahriche, K. L. McDonald, S. Nasri and I. Picek, arXiv:1603.01247 [hep-ph].
- [105] T. Nomura, H. Okada and Y. Orikasa, arXiv:1602.08302 [hep-ph].
- [106] P. Jean *et al.*, Astron. Astrophys. **407**, L55 (2003) doi:10.1051/0004-6361:20031056 [astro-ph/0309484].
- [107] D. Tucker-Smith and N. Weiner, Phys. Rev. D **64**, 043502 (2001) doi:10.1103/PhysRevD.64.043502 [hep-ph/0101138].
- [108] M. Cirelli, N. Fornengo and A. Strumia, Nucl. Phys. B **753**, 178 (2006) [hep-ph/0512090].
- [109] M. Cirelli, A. Strumia and M. Tamburini, Nucl. Phys. B **787**, 152 (2007) [arXiv:0706.4071 [hep-ph]].
- [110] K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014).
- [111] J. A. Casas and A. Ibarra, Nucl. Phys. B **618**, 171 (2001) [hep-ph/0103065].
- [112] T. Toma and A. Vicente, JHEP **1401**, 160 (2014) [arXiv:1312.2840, arXiv:1312.2840 [hep-ph]].
- [113] CMS Collaboration [CMS Collaboration], CMS PAS HIG-14-039.
- [114] M. Benayoun, P. David, L. Delbuono and F. Jegerlehner, Eur. Phys. J. C **72**, 1848 (2012) [arXiv:1106.1315 [hep-ph]].
- [115] K. Griest and D. Seckel, Phys. Rev. D **43**, 3191 (1991).
- [116] P. A. R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. (2014) [arXiv:1303.5076 [astro-ph.CO]].
- [117] J. Knodlseder *et al.*, Astron. Astrophys. **411**, L457 (2003) [astro-ph/0309442].
- [118] S. Khalil and O. Seto, JCAP **0810**, 024 (2008) [arXiv:0804.0336 [hep-ph]].
- [119] CMS Collaboration [CMS Collaboration], CMS-PAS-EXO-15-004.
- [120] The ATLAS collaboration, ATLAS-CONF-2015-081.
- [121] S. Kanemura, K. Nishiwaki, H. Okada, Y. Orikasa, S. C. Park and R. Watanabe, arXiv:1512.09048 [hep-ph].

- [122] T. Nomura and H. Okada, Phys. Lett. B **755**, 306 (2016) doi:10.1016/j.physletb.2016.02.022 [arXiv:1601.00386 [hep-ph]].
- [123] J. H. Yu, arXiv:1601.02609 [hep-ph].
- [124] R. Ding, Z. L. Han, Y. Liao and X. D. Ma, arXiv:1601.02714 [hep-ph].
- [125] T. Nomura and H. Okada, arXiv:1601.04516 [hep-ph].
- [126] H. Okada and K. Yagyu, arXiv:1601.05038 [hep-ph].
- [127] T. Nomura and H. Okada, arXiv:1601.07339 [hep-ph], accepted for publication in Physics Letters B.
- [128] P. Ko, T. Nomura, H. Okada and Y. Orikasa, arXiv:1602.07214 [hep-ph].
- [129] A. E. C. Hernandez, arXiv:1512.09092 [hep-ph].
- [130] C. Arbelaez, A. E. C. Hernandez, S. Kovalenko and I. Schmidt, arXiv:1602.03607 [hep-ph].
- [131] W. Chao, arXiv:1512.08484 [hep-ph].